Int. J. Multiphase Flow Vol. 19, No. 6, pp. 1153–1158, 1993
Printed in Great Britain

0301-9322/93 $6.00 + 0.00
Pergamon Press Ltd

NONLINEAR INSTABILITY DURING THE ISOTHERMAL DRAW OF OPTICAL FIBERS

P. GOSPODINOV and VL. ROUSSINOV
Institute of Mechanics and Biomechanics, Bulgarian Academy of Sciences, Acad. G. Bonchev Bl. 4, 1113 Sofia, Bulgaria

(Received 15 October 1991; in revised form 11 August 1993)

Abstract—The nonlinear instability of the isothermal draw of optical fibers from cylindric preforms is studied. The unsteady model of the process is solved numerically, accounting for the effects of inertia, gravity and surface forces. The effect of viscosity and gravity on the nonlinear stability of the process is studied. The possibility of draw resonance occurring is shown for a rate ratio much lower than the critical one, obtained when solving the simplified model. The proposed solution can be used to study technological stability and to model the draw of fibers of other materials which behave as Newtonian fluids.

Key Words: nonlinear stability, optical fibers, draw, spinning

1. INTRODUCTION

The draw of optical glass fibers from cylindric preforms or melt is characterized by mechanical disturbances which occur in the technological installation. They cause unevenness of the fiber radius length and velocity profile.

A quasi one-dimensional model of the isothermal flow of a Newtonian fluid is considered when modelling the material behavior. A simplified model of draw is used and inertia and gravity, as well as surface tension, are usually disregarded. The equations, describing the time-dependent behavior of small disturbances in a steady state, have been derived and analyzed by Pearson & Matovich (1969), while Yarin (1987) gives numerical and asymptotic solutions. The draw of polymer fibers from melt has been analyzed by Ishihara & Kase (1975, 1976). They solve the unsteady problem numerically and study the draw resonance under the same simplifications and by using an explicit scheme. Berman & Yarin (1983) propose an analytical solution and determine the critical value of the draw ratio.

The effect of gravity and viscous and surface forces on the draw linear instability is studied by Shah & Pearson (1972).

2. MATHEMATICAL MODEL

The study of the draw of glass fibers from melt and from cylindric preforms, accounting for inertia and gravity and surface tension, and considering isothermal draw conditions, requires the solution of the unsteady equations of motion and continuity. For vertical draw they have the following form:

$$\rho r^2\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = \frac{\partial}{\partial x}(r^2 \cdot \Sigma_{xx}) + \rho g r^2 + \sigma \frac{\partial r}{\partial x} \qquad [1]$$

and

$$\frac{\partial r^2}{\partial t} + \frac{\partial}{\partial x}(r^2 v) = 0, \qquad [2]$$

where

$$\Sigma_{xx} = 3\eta \frac{\partial v}{\partial x} \qquad [3]$$

is the viscous stress tensor.

The following notations are used: t—time; x—coordinate along the fiber symmetry axis; r—fiber radius; v—velocity along x; ρ and η—density and viscosity of the drawn vitreous mass; g—Earth's acceleration; σ—coefficient of surface tension.

The sign "+" before the term containing g in [1] reflects the case when the direction of the draw and feed velocity coincides with that of gravity.

Equations [1] and [2] are given in a dimensionless form by using the following scales: for time—L/v_1; for velocity—v_1/E; for the fiber radius—$r_0/E^{1/2}$; and for the x-coordinate—L. The feed and draw velocities at the cross section $x = 0$ and $x = L$ are denoted by v_0 and v_1, while the draw ratio is denoted by $E = v_1/v_0$. L is length of the drawn fiber ($0 \leqslant x \leqslant L$) and r_0 (at $x = 0$) is the preform or nozzle radius.

The dimensionless velocity and fiber radius are denoted by V and R, while the notations for the coordinate x and time t are kept the same. Thus, the dimensionless equations of the model are obtained:

$$R^2\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right) = \frac{3(N \cdot E)^{1/2}}{\text{Re}}\frac{\partial}{\partial x}\left(R^2\frac{\partial V}{\partial x}\right) + \frac{1}{\text{Fr} \cdot (N \cdot E)^{1/2}}R^2 + \frac{E^{1/2}}{\text{We}}\frac{\partial R}{\partial x} \qquad [4]$$

and

$$\frac{\partial R^2}{\partial t} + \frac{\partial}{\partial x}(R^2 V) = 0. \qquad [5]$$

The following criteria of similarity are used in [4] and [5]: $\text{Re} = (\rho v_1 r_0)/\eta$—the Reynolds number; $\text{Fr} = v_1^2/(g r_0)$—the Froude number; and $\text{We} = (\rho r_0 v_1^2)/\sigma$—the Weber number. $N = r_0^2/(L^2 E)$ is a dimensionless parameter, characterizing the ratio between the transversal and longitudinal dimensions.

The initial and boundary conditions are given by

$$R(0, t) = R_0 = E^{1/2} \quad \text{for } x = 0, \qquad [6]$$

$$V(0, t) = V_0 E \quad \text{for } x = 0, \qquad [7]$$

$$V(1, t) = V_1 = 1 \quad \text{for } x = 1 \qquad [8]$$

and

$$R[x, 0] = \varphi_1(x), \quad V(x, 0) = \varphi_2(x) \quad \text{for } t = 0. \qquad [9]$$

3. NUMERICAL SOLUTION

The unsteady model is treated numerically by using an implicit scheme of "Crank–Nicolson" type. Symmetric approximations of the derivatives $\partial V/\partial x$, $\partial R/\partial x$ and $\partial^2 V/\partial x^2$ are used, where:

$$\frac{\partial V}{\partial x} \sim \frac{V_{i+1} - V_{i-1}}{2 \cdot \Delta x}, \quad \frac{\partial R}{\partial x} \sim \frac{R_{i+1} - R_{i-1}}{2 \cdot \Delta x}, \quad \frac{\partial^2 V}{\partial x^2} \sim \frac{V_{i+1} - 2V_i + V_{i-1}}{\Delta x^2}.$$

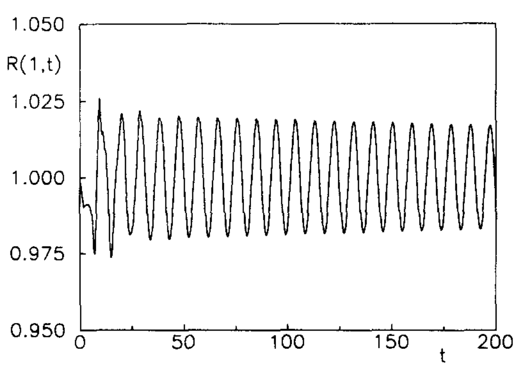

Figure 1. Self-sustained oscillations when surface forces are disregarded.

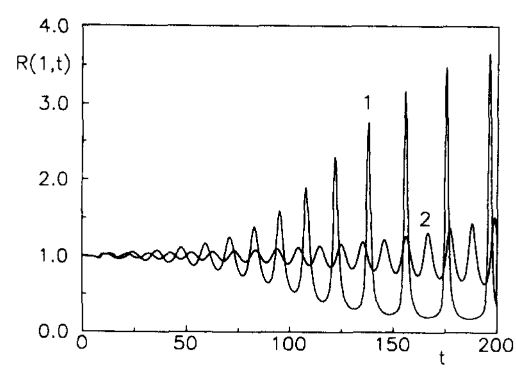

Figure 2. Draw resonance: curve 1—accounting for gravity; curve 2—disregarding gravity.

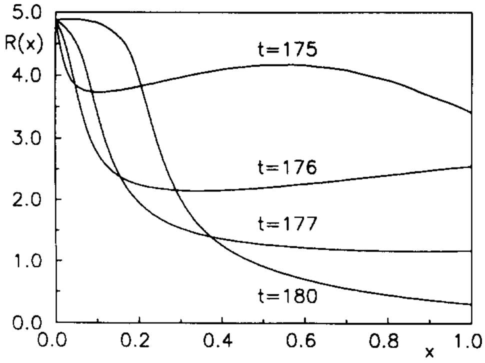

Figure 3. Running waves along the surface of the formed fiber.

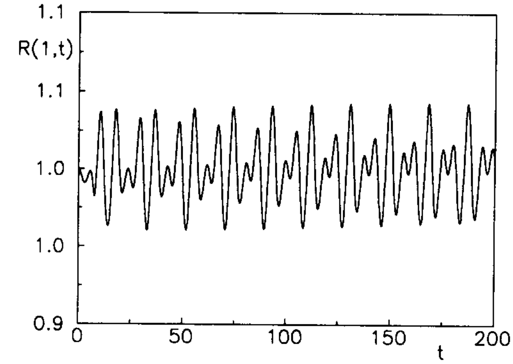

Figure 4. Development of forced harmonic disturbances in the draw velocity for a critical draw ratio.

Here $R_i = R(x_i, t)$, $V_i = V(x_i, t)$, $x_i = (i - 1) \cdot \Delta x$ and i denotes the number of a point within the discretized interval [0, 1].

The time derivatives of R and V are approximated as

$$\frac{\partial}{\partial t}(R(x_i, t)) \sim \frac{R_i - \check{R}_i}{\Delta t}, \quad \check{R}_i = R(x_i, t - \Delta t);$$

where Δx and Δt are steps of variation of the dimensionless coordinates x and t. Due to the nonlinearity with respect to R and V, an iteration procedure with a time step Δt is used. The procedure terminates when a definite accuracy is attained.

The initial distributions $\varphi_1(x)$ and $\varphi_2(x)$, obtained during the numerical solution of the steady equations and corresponding to the initial model [1]–[7], are given at the moment $t = 0$.

A Fortran program is designed to solve the problem, where calculations with double precision are performed on a 486/50 MHz PC. A scheme with 200 steps along the spatial coordinate x is used and the time step Δt is determined after performing numerical experiments.

4. NUMERICAL RESULTS

The proposed solution can be used for the study of nonsteady processes, where the following conditions can subsequently substitute for either of [6] or [7] for $x = 0$, or [8] for $x = 1$:

$$R(0, t) = R_0(1 + \delta R), \qquad [10]$$

a stepwise disturbance of the preform initial radius;

$$R(0, t) = R_0[1 + \delta R \cdot \sin(\omega_1 t)], \qquad [11]$$

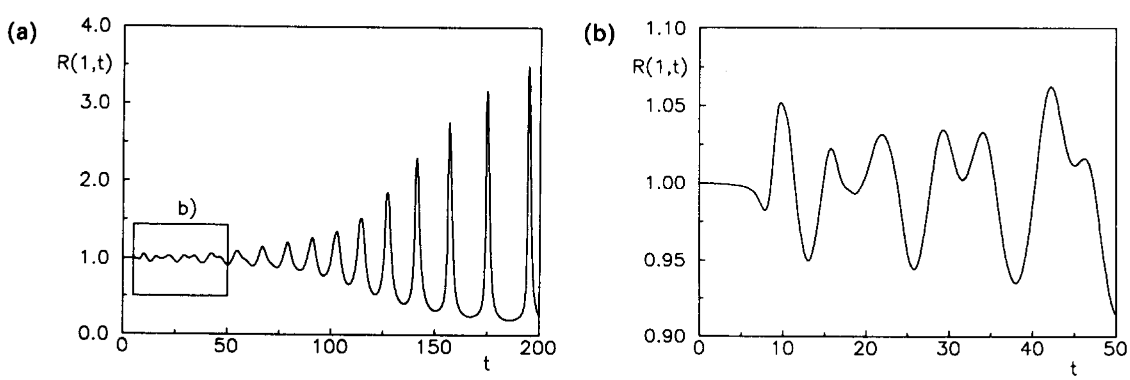

Figure 5. Draw resonance for harmonic disturbances of the preform radius: (a) $0 \leqslant t \leqslant 200$; (b) $0 \leqslant t \leqslant 50$.

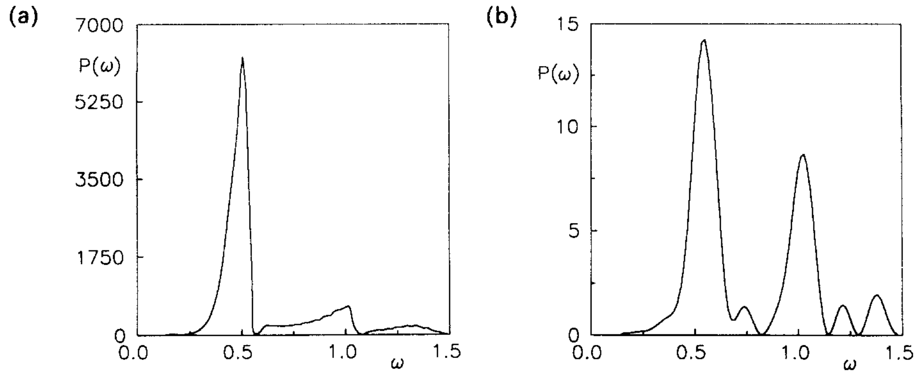

Figure 6. Spectral power corresponding to (a) figure 5(a) and (b) figure 5(b).

a sinusoidal disturbance of the preform initial radius;

$$V(1, t) = V_1(1 + \delta V), \quad [12]$$

a stepwise disturbance of the draw velocity; or

$$V(1, t) = V_1[1 + \delta V \cdot \sin(\omega_1 t)], \quad [13]$$

a sinusoidal disturbance of the draw velocity;

Draw velocity disturbances are given when studying self-sustained oscillations, while conditions [6] and [7] are kept. Moreover, condition [12], with $\Delta V = 0.05$, is given for $0 \leqslant t \leqslant 2$ instead of condition [8]. The frequency of the forced harmonic disturbances in [11]–[13] is denoted by ω_1.

Depending on the draw ratio E, the amplitudes of the occurring oscillations decrease or grow. Self-sustained oscillations with a constant amplitude correspond to the critical draw ratio. The value $E_* = 20.22$, obtained by performing numerical experiments and disregarding the viscous, surface and gravity forces, agrees with the results of the simplified solution (Pearson & Matovich 1969; Gelder 1971; Berman & Yarin 1983).

In the real case, the critical draw ratio and the corresponding circular frequency ω are affected by the viscosity of the drawn vitreous mass and by the surface, gravity and inertia forces. Our numerical results are obtained on the basis of the following initial data: $\rho = 3000\ \text{kg/m}^3$, $\sigma = 0.25$ N/m. The draw velocity is $v_1 = 0.05$ m/s and the preform initial radius is $r_0 = 0.005$ m. The corresponding dimensionless parameters are Fr = 0.051, We = 0.15 and $L/r_0 = 50$. The viscosity of the vitreous mass is assumed to be within the ranges $\eta = 10^7$ to $10^2\ \text{N} \cdot \text{s/m}^2$, in accordance with Doremus (1973), Paek & Kurkjian (1975) and Paek & Runk (1978). The following values of Re correspond to these values: from $\text{Re} = 7.5 \times 10^{-8}$ to 7.5×10^{-3}.

Values of the critical draw ratio and circular frequency are found for some characteristic viscosity values (in $\text{N} \cdot \text{s/m}^2$) by performing a numerical experiment:

$$\eta = 10^7, \quad E = 20.25, \quad \omega = 0.6903;$$

$$\eta = 10^6, \quad E = 20.55, \quad \omega = 0.6903;$$

$$\eta = 10^5, \quad E = 23.87, \quad \omega = 0.6673;$$

$$\eta = 10^4, \quad E = 85.30, \quad \omega = 0.5400.$$

The results, given in figures 1–6, are obtained for a viscosity value $\eta = 10^5\ \text{N} \cdot \text{s/m}^2$ ($\text{Re} = 7.5 \times 10^{-6}$).

The change in $R(1, t)$ is shown in figure 1, for self-sustained oscillations with a draw ratio $E = 23.87$. The surface forces are disregarded, i.e. $\sigma = 0$.

The results given in figure 2 (curve 1) are obtained on the basis of the assumption that the draw velocity is opposite to gravity (i.e. the sign in front of the term containing g in [1] is a minus). The numerical data for curve 2 are obtained after disregarding gravity, i.e. $g = 0$. The circular frequencies for both cases are equal to $\omega_1 = 0.3451$ and $\omega_2 = 0.5983$, respectively. The oscillations are with increasing amplitude, which for curve 1 attains a value several times larger than the steady radius value at $x = 1$. The comparison between the two curves shows that if gravity is opposite to the draw direction, larger amplitudes and smaller circular frequencies of resonance oscillations occur.

Figure 3 shows the form of $R(x)$ for some fixed time values. It is obtained under the same conditions as those in figure 2 (curve 1). The wave on the fiber surface is nonsteady and running along x. The change in the vertical draw direction yields not only draw resonance, but also a significant change in the shape and dimensions of the formed fiber (the latter is bounded by sections $x = 0$ and $x = 1$).

It is of practical interest to study the solution stability for a periodic disturbance of draw velocity, caused by an imperfection in the draw mechanism. Condition [13] is used in this case. The development of $R(1, t)$ for $\omega_1 = 1$, $\delta V = 0.05$, is given in figure 4. The frequency analysis shows two prevailing frequencies: the first one coincides with that of the external forced disturbances; while the second is $\omega_2 = 0.675$—the frequency of the self-sustained oscillations.

Periodic disturbances during draw from cylindric preforms can also be introduced as a result of an inaccuracy of the preform radius. This effect corresponds to condition [11] of the mathematical model. The development of forced periodic disturbances of the initial radius, with a frequency $\omega_1 = 1$ and amplitude $\Delta R = 0.005$, is shown in figure 5(a); therein draw is opposite to gravity. Two frequencies with a close spectral power are observed for time values $0 \leqslant t \leqslant 50$—figure 5(b). These are the frequency of forced disturbances and the eigenfrequency $\omega = 0.5522$. The oscillations assume a character of draw resonance with time, where the eigenfrequency prevails. The results of the frequency analysis are shown in figure 6. The frequency spectral power is denoted by $P(\omega)$ (Angot 1957).

The numerical experiments confirm the conclusions of Shah & Pearson (1972) regarding the effect of gravity and surface forces on draw stability. As the calculations show, the draw velocity direction is essential for the stability of the vertical draw. Values of the critical draw ratio are listed below. They are obtained for different values of the viscosity of the vitreous mass and for the vertical draw (opposite to gravity). The rest of the parameters, included in the dimensionless model, are the same:

$$\eta = 10^7, \quad E = 20.11, \quad \omega = 0.6903;$$

$$\eta = 10^6, \quad E = 19.78, \quad \omega = 0.6903;$$

$$\eta = 10^5 \quad E = 16.88, \quad \omega = 0.7210;$$

$$\eta = 10^4, \quad E = 14.33, \quad \omega = 0.7517;$$

$$\eta = 10^3, \quad E = 4.993, \quad \omega = 0.8437.$$

5. CONCLUSIONS

An algorithm for the analysis of draw nonlinear instability is proposed. It is based on an implicit scheme and considers the draw of fibers from cylindric preforms and melts. It also accounts for inertia, gravity and surface tension. The numerical solution allows for the estimation of limits, within which it is admissable to violate values of the initial radius, draw or feed velocity. Viscosity and gravity effects on the draw instability are also studied.

The results show that resonance can occur for a draw ratio, significantly lower than the critical one, obtained by using the simplified model. The proposed solution can be used to model the draw of fibers of other materials which behave as Newtonian fluids.

REFERENCES

Angot, A. 1957 A l'usage des ingenieurs de l'elektrotechnique et des telecommunications. Paris.

Berman, V. S. & Yarin, A. L. 1983 Dynamic regimes of fiber forming. *Mech. Liquids Gases* **6,** 31–37 (in Russian).

Doremus, R. H. 1973 *Glass Science*. Wiley, New York.

Gelder, D. 1971 The stability of fiber drawing processes. *Ind. Engng Chem. Fundam.* **10,** 534–535.

ISHIHARA, H. & KASE, S. 1975 Studies of melt spinning. V. Draw resonance as a limit cycle. *J. Appl. Polym. Sci.* **19,** 557–566.

ISHIHARA, H. & KASE, S. 1976 Studies of melt spinning. VI. Simulation of draw resonance using Newtonian and power law viscosities. *J. Appl. Polym. Sci.* **20,** 169–191.

PAEK, U. C. & KURKJIAN, C. R. 1975 Calculation of cooling rate and induced stresses in drawing of optical fibers. *J. Am. Ceram. Soc.* **58,** 330–335.

PAEK, U. C. & RUNK, R. B. 1978 Physical behaviour of the neck down region during furnace drawing of silica fibers. *J. Appl. Phys.* **49,** 4417–4422.

PEARSON, J. R. A. & MATOVICH, M. A. 1969 Spinning a molten thread line. *Ind. Engng Chem. Fundam.* **8,** 605–609.

SHAH, Y. T. & PEARSON, J. R. A. 1972 On the stability of fiber spinning—general case. *Ind. Engng Chem. Fundam.* **11,** 150–153.

YARIN, A. L. 1987 On the occurrence of auto-oscillations during fiber forming. *Appl. Math. Mech.* **47,** 82–88 (in Russian).